Math for Robots

You thought there wasn't going to be mathematics?

Why is it important?

- So your robot doesn’t brake!
- Lese likely to burn out motors.
- Practise engineering skills.
- Impress the judges.

Disclaimer

We are going to be looking simplified situations.

This means that a real world situation may act differently.

Wheels

What do Wheels change?

Speed or Velocity

Circumference of the wheel (circle)

TETRIX® MAX Wheels – diameter 3in or 4in

$$C = \pi \times d$$

$$C_1 = 3.14 \times 3in = 9.42in$$

$$C_2 = 3.14 \times 4in = 12.46in$$

Velocity

REV HD Hex Motor Free speed N = 150 rpm

$$V = C \times N$$

$$V_1 = 9.42in \times 150 \frac{rev}{min} = 1413 \frac{in}{min} \times \frac{1\,min}{60sec} = 23.55 \frac{in}{sec} = 1.96 \frac{ft}{sec}$$

$$V_2 = 12.46in \times 150 \frac{rev}{min} = 1869 \frac{in}{min} \times \frac{1\,min}{60sec} = 31.15 \frac{in}{sec} = 2.59 \frac{ft}{sec}$$

Percent Increase

$$percent\ increase = \frac{New\ Value - Old\ Value}{Old\ Value} \times 100\%$$

$$percent\ increase = \frac{2.59 - 1.96}{1.96} \times 100\% = 32\%\ increase\ in\ speed$$

How to change the Speed?

- Gearing down is when a larger gear drives a smaller gear. This will increase the speed but lower the torque.
    - Gearing down will have a gear ration less then 1
- Gearing up is when a smaller gear drives a larger gear. This lower the speed but increase the torque.
    - Gearing up will have a gear ration greater then 1

Gearing

$$Gear\ Ratio = \frac{Driven\ Gear\ Teeth}{Driving\ Gear\ Teeth}$$

$$Gear\ Ratio = \frac{45t}{72t} = 0.625$$

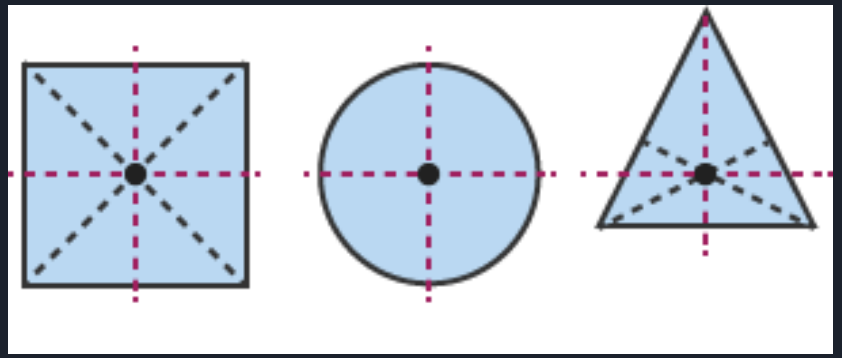
$$Output\ Speed = \frac{Input\ Speed}{Gear\ Ratio}$$

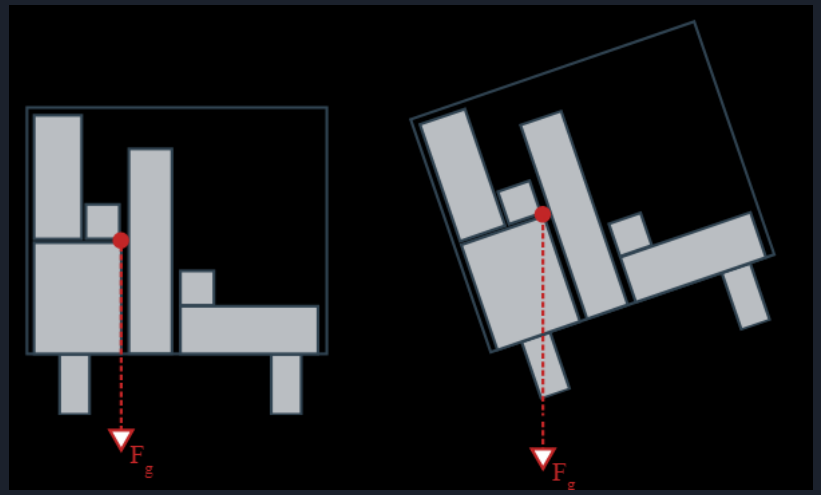
$$Output\ Speed = \frac{2.59\frac{ft}{sec}}{0.625} = 4.14\frac{ft}{sec}$$

$$percent\ increase = \frac{4.14 - 2.59}{2.59} \times 100\% = 60\%\ increase\ in\ speed$$

Centre of Mass

Center of Mass





Motors

Selecting a motor

What information is needed to select a motor?

- Speed
- Torque
- Power
- Electrical requirements

All this information is provide on datasheet or website

REV HD Hex Motor http://www.revrobotics.com/rev-41-1301/

Example

Assume a robot (on earth) has a mass of 42 lb and is being lifted 4in.

What is the work needed to complete is task? What is the power if

the lift happens in 5 seconds?

Conversations

- Mass 42 lb = 19.05kg ≈ 20kg
- Distance = 4in = 10.16cm = 0.1016m

How to select a motor for an Arm?

Find the power is needed to complete the task. Think big picture.
Do we have all the unknowns?

- Mass - mass of the object being moved or lifted
- Acceleration - On Earth? assume 9.81 [m/s^2]
- Distance - How far is the object moving?
- Time - How fast does this move happen?

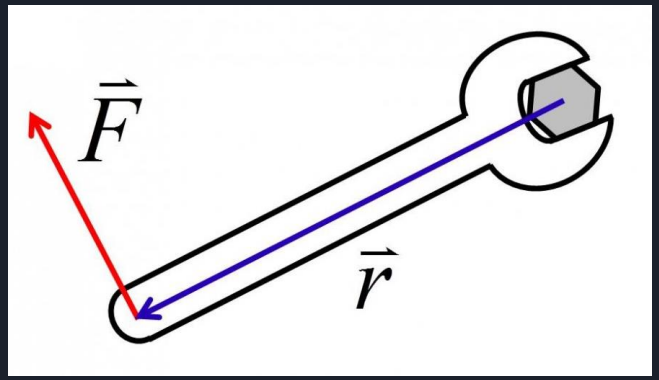
Definitions

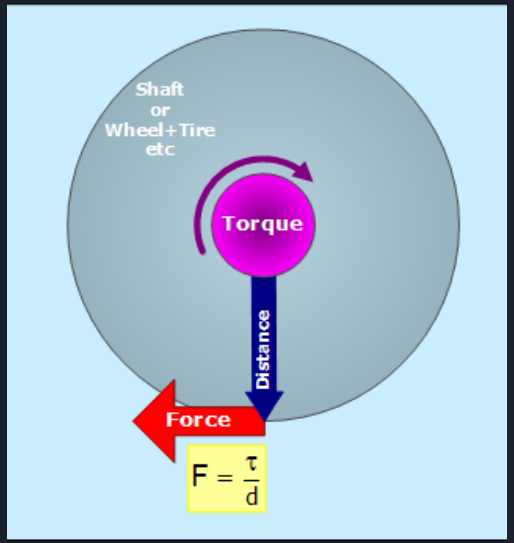
Mass (m) is used to measure the amount of matter in an object. Often expressed in units of [kg] or [lb].

Acceleration (a) is used to describe how fast a object is changing. Expressed in units of [m/s^2] or [in/s^2].

Force (F) is the potential for an object to do work. Often expressed in units of [N].

Torque (T) is roughly quantifies the turning force on an object like a gear or a wheel. Torque is commonly expressed in units of [Nm], [oz·in], or[in·lbs].





Work (W) is used to describe changes in energy. Work is independent of the path taken and is defined as force times displacement. For example if a 1 [kg] weight is lifted vertically 1 [m] against gravity at a constant velocity the work done is 1[kg]*9.8[m/s^2]*1[m] = 9.8 [kg*m^2/s^2] or 9.8 joules[J]. But joules are also [Nm].

Power (P) is the rate of work over time. The difference between power and work it takes the same amount of work to carry a brick up a mountain whether you walk or run, but running takes more power because the work is done in a shorter amount of time. The SI unit for power is the Watt (W) which is equivalent to one joule per second (J/s).

$$Force = Mass \times Acceleration$$

$$F = m \times a = m \times g$$

$$F = 20kg \times 9.81\frac{m}{s^2} = 196.2N$$

$$Work = Torque = Force \times Distance$$

$$W = T = F \times d$$

$$W = 196.2N \times 0.1016m = 19.93J$$

$$or\ T = 19.93Nm$$

Note: The equations are for one dimension.

$$Power = \frac{Work}{Time}$$

$$P = \frac{W}{t}$$

$$P = \frac{19.93J}{5s} = 3.98W\ \approx 4W$$

With a safety factor of 2

$$P \times 2$$

$$P \times 2\ = 4 \times 2 = 8\ W$$

Dose the motor work?

Look back at the datasheet.

REV HD Hex Motor http://www.revrobotics.com/rev-41-1301/

Check both the Torque[Newton meters] and the Power [Watts]

- If the motor dose not have enough power, pick a different motor.
- If the motor dose not have the correct Stall Torque, use gearing.

Gearing

$$Gear\ Ratio = \frac{Driven\ Gear\ Teeth}{Driving\ Gear\ Teeth}$$

$$Output\ Torque = Input\ Torque\ \times Gear\ Ratio$$

$$Input\ Torque = \frac{Output\ Torque}{Gear\ Ratio}$$

$$Gear\ Ratio = \frac{86t}{28t} \times \frac{86t}{42t} = 6.29$$

$$Output\ Torque = 4.2\ Nm \times 6.29 = 26.41\ Nm$$

Consider using a Safety Factor of 1.5-2

Note: Gear teeth numbers are metal gears from REV.

Links for more information

- Work, Energy, and Power: Crash Course Physics #9
- Forces - Newton's Laws: Crash Course Physics #5
- VEX IQ Mechanisms - Gear Ratio